Minimum Latency Broadcast Scheduling in Single-Radio Multi-Channel Wireless Ad-Hoc Networks

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***Abstract*—We study the minimum latency broadcast scheduling (MLBS) problem in Single-Radio Multi-Channel (SR-MC) wireless ad-hoc networks (WANETs), which are modeled by Unit Disk Graphs. Nodes with this capability have their fixed reception channels, but can switch their transmission channels to communicate with their neighbors. The single-radio and multi-channel model prevents existing algorithms for single-channel networks achieving good performance. First, the common assumption that one transmission reaches all the neighboring nodes does not hold naturally. Second, the multi-channel dimension provides new opportunities to schedule the broadcast transmissions in parallel. We show MLBS problem in SR-MC WANETs is NP-hard, and present a benchmark algorithm: Basic Transmission Scheduling (BTS), which has approximation ratio of $4k + 12$. Here k is the number of orthogonal channels in SR-MC WANETs. Then we propose an Enhanced Transmission Scheduling (ETS) algorithm, improving the approximation ratio to $k + 23$. Simulation results show that ETS achieves better performance over BTS, and the performance of ETS approaches the lower bound.**

I. Introduction

Single-Radio Multi-Channel (SR-MC) wireless ad hoc networks (WANETs) have gained significant attentions in the past few years because of their great promise of low cost, high throughput and spectral efficiency. By using multiple orthogonal channels in single radio, we can enhance spatial reuse [1], alleviate jamming attack [2], and enable dynamic access to the scarce spectrum resource [3]. Several typical multi-channel MACs [4]–[6] are proposed to fully utilize the single-radio multi-channel capability.

Broadcast is a fundamental operation in wireless networks for routing discovery, information dissemination, and so on. Here we focus on the minimum latency broadcast scheduling operation, where broadcast latency is defined as the end-to-end latency by which all nodes in the network receive the broadcast message from source node. Such concern is very important in various applications such as military communications, disaster relief and rescue operations.

For some SR-MC networks [4], broadcast packets can be delivered in a dedicated control channel (DCC). However, the DCC can be a bottleneck, vulnerable to jamming attack [2], and even unavailable [3]. Therefore, in this paper, we consider SR-MC WANETs without the DCC. Given no DCC, the solutions to MLBS problem in single-channel WANETs [9], [10] cannot work directly, because single transmission cannot reach all the neighboring nodes if the radios of neighboring nodes are tuned to different channels. In other words, it may cost multiple transmissions to deliver a message to all its neighbors. This property is similar to the *partial broadcast property* in duty-cycled WANETs [12], [13]. However, parallel transmissions can still happen in different nodes within several channels at the same time (we refer this property as *multi-partial broadcast property*), which is not allowed in duty-cycled networks. Hence, solutions from duty-cycled networks cannot achieve best performance, and can be further optimized. On the other hand, compared with solutions for MR-MC WANETs [14], single-radio mode does not allow a node to transmit simultaneously in several channels, resulting in less parallel transmission opportunities. Therefore, *multi-partial broadcast property* brings a new challenge for designing efficient, collision-free broadcast protocols.

In this paper, we investigate the minimum latency broadcast scheduling (MLBS) problem in SR-MC WANETs. We first show such problem is NP-Hard, and then design efficient algorithms with performance guarantees. To solve the problem, we construct a Shortest-Path Tree (SPT), and schedule the transmissions layer by layer. By utilizing the *multi-partial broadcast property*, we can schedule the cross-layer and same-layer transmissions with polynomial-time complexity. Our main contributions are summarized as follows:

- We show that a Basic Transmission Scheduling (BTS) algorithm with approximation ratio of 4k+12 can be obtained by modifying existing approaches properly, where k is the number of available orthogonal channels.
- We present an Enhanced Transmission Scheduling (ETS) algorithm by utilizing the parallel transmission opportunities, which has an improved approximation ratio of $k + 23$. The performance is evaluated through extensive simulations.

The rest of the paper is organized as follows. Section II gives the related work. Network model and problem statement are presented in Section III. We first propose BTS in Section IV, and give ETS in Section V. Then we validate our result by simulations in Section VI. Section VII concludes our paper.

II. Related Works

Channel assignment in SR-MC WANETs is the most related works to ours. In general, there are three kinds of channel assignment approaches: *fixed*, *semi-dynamic* and *dynamic*. In fixed channel assignment method, nodes are assigned fixed channels for permanent use, and radios do not change the operating frequency. In *semi-dynamic* approaches, though the assigned reception channel is fixed, nodes can still change their transmission channel to communicate with neighbors that have different reception channels. In dynamic approaches, nodes are not assigned static channels, and can switch their channel dynamically according to a pre-defined rule, e.g., quorum sequences [6]. Moreover, some works consider channel assignment and other problems jointly, e.g., minimizing interference [7], fast data dissemination [8]. In contrast, here we consider the minimum latency broadcast scheduling problem after channel assignment, and assume *semi-dynamic* strategy.

Collision-free minimum latency broadcast scheduling is well studied in single-channel WANETs. Gandhi et al. [9] show MLBS problem in UDGs is NP-hard. Recently, Huang et al. [11] gives an algorithm with approximation ratio of 12. For duty-cycle WANETs, Hong et al. [12] shows MLBS to be NP-hard too, and present an algorithm with approximation ratio $24|T|+1$ where $|T|$ is the length of one scheduling period. Our SR-MC scenario has the multi-channel dimension, which is not considered in single-channel and duty-cycled WANETs. Qadir et al. [14] propose several algorithms for minimum latency broadcasting in MR-MC, multi-rate wireless meshes. However, the proposed algorithms depend on the multi-radio capability (i.e., multi-connection links), and all heuristic algorithms are evaluated by simulations without theoretical analysis. To the best of our knowledge, [15] is the only paper to consider minimal latency broadcast directly in multi-channel cognitive radio networks, which is very close to us. The key difference is that we allow channel switch while [15] assumes not. Moreover, [15] shows the closeness of their solution to the optimal solution through simulations. Instead, we give two algorithms with performance guarantees.

III. Preliminaries

A. Network Model and Assumptions

The SR-MC WANETs can be modeled as a Unit Disk Graph (UDG) $G = (V, E)$, where V is the set of nodes ($|V| = n$), and E is the set of links. An edge $\{u, v\} \in E$ iff. u and v is within each other's communication range. We also assume that the time is slotted. Each time slot is equal length, and long enough for one packet transmission and reception. Moreover, the slot boundary is almost aligned, which can be achieved by local synchronization protocols. We further assume that reception is error-free if no collision happens, which is quite accurate because control packets are often well protected by physical layer, e.g., minimum data rate 6 Mbps in IEEE 802.11 a/g standard. Both synchronization and error-free assumptions are widely adopted by previous works [9]–[11], [14], [15].

The SR-MC WANETs have a total of k orthogonal channels denoted by $C = \{1, 2, ..., k\}$, and each node is equipped with only one radio. The radio interface can be set on any channel to transmit or listen, but not simultaneously. The reception channel is chosen randomly from C during network initialization, which can be defined using a channel assignment function A. For $v \in V$, $A(v) = c$ where $c \in C$. Neighboring nodes may have different reception channels. In order to enable connectivity, we assume that transmission nodes can switch their channels to set up connections. Note that in $G(V, E)$, the edge definition depends on topology instead of channel since we allow channel switch. We also assume that the neighbors' reception channels are known beforehand, which is often achieved during neighbor discovery.

B. Problem Statement

Here we consider the single-source broadcast problem. Suppose the source node is s, and the broadcast task completes when all the other nodes receive messages sent from s. Assume s starts the broadcast operation at time-slot 1. Then we formulate the MLBS problem (decision version) in SR-MC WANETs as follows (MLBS-SRMC): *Given a UDG $G(V, E)$ with channel assignment function A, and positive integer T, is there an assignment of time slots and transmission channels to nodes $v \in V$, such that the broadcast scheduling is collision-free and the schedule length is no more than T?*

Theorem 1. *The MLBS-SRMC problem is NP-hard.*

Proof: We prove this theorem using restriction technique [17]. If we restrict function A to map to one single channel, our problem is exactly the MLBS problem in single-channel WANETs, which is NP-hard [9]. Hence MLBS-SRMC problem is NP-hard. ∎

Our objective can be interpreted as finding a broadcast schedule $S = \{S_1, ..., S_T\}$, where S_i, $1 \leq i \leq T$, is the set of transmitting instances at time slot i, i.e. $S_i = \{(v_0, c_0), ..., (v_j, c_j)\}, j \geq 0$. At time slot 1, only s can transmit. After T time slots, all nodes in V receive messages from s. Our problem can be converted to minimize T. Note that if we set the cost of each edge one unit, we can construct a Shortest-Path tree (SPT) rooted s. Then the lower bound for broadcast is the depth of SPT denoted by l, i.e., $T_{min} >= l$.

C. Graph-Theoretic Definitions and Results

Let $G = (V, E)$ be an undirected UDG. The subgraph of G induced by a subset U of V is denoted by $G[U]$. The k-th power of G, denoted by G^k, is a graph over V in which there is an edge between two nodes u and v if and only if their distance is G is at most k. The minimum degree of G is denoted by $\delta(G)$. The inductility of G is defined by $\delta^*(G) = \max_{U \subseteq V} \delta(G[U])$. $\delta^*(G) \leq 11$ for UDG [10]. It's well-known that the node coloring of G induced by a smallest-degree-last ordering uses at most $1 + \delta^*(G)$ colors [16]. An Independent Set (IS) of a graph G is a set of vertices in G that no two of which are adjacent. A maximal independent set of G is not a subset of any other IS of G. Each node in V can be adjacent to at most five nodes in any IS of $G(V, E)$ [10], and can have at most nineteen two-hop neighbors in any IS of $G(V, E)$ [11].

TABLE I
TERMINOLOGY

Symbol	Definition
k	number of available orthogonal channels in $G(V, E)$
$N(u)$	neighboring set of nodes u in $G(V, E)$
T_{SPT}	Shortest-Path Tree rooted s in $G(V, E)$
l	depth of T_{SPT}
L_i	nodes of layer i in T_{SPT}
H_c	nodes using channel c in T_{SPT}
$L_{i,c}$	nodes of layer i using channel c in T_{SPT}, $L_{i,c} = L_i \cap H_c$
$M_{i,c}$	maximal independent set (dominators) of $L_{i,c}$
$P(S)$	set of parent nodes of set S in T_{SPT}
$P_{i,c}$	parent nodes (connectors) of $M_{i,c}$ which are selected greedy
T_b	broadcast tree constructed by Algorithm 2, including nodes V, edges E and cover function C

IV. BASIC TRANSMISSION SCHEDULING

In this section, we give an algorithm Basic Transmission Scheduling (BTS) for minimum latency broadcast scheduling problem in UDGs, which is a simple extension to existing approaches [10], [12]. The main notations used in this paper are summarized in Table I.

A. Algorithm Description

Let L_i be nodes of layer i in T_{SPT}, $i = 0, 1, ..., l$, H_c be nodes using channel c in G, $c = 1, 2, ..., k$, and $L_{i,c}$ be nodes in layer i using channel c ($L_{i,c} = L_i \cap H_c$). Then, BTS can find a maximal independent set $M_{i,c}$ for each $L_{i,c}$ by adding eligible nodes sequentially. Let $P(S)$ be the set of parent nodes of S in T_{SPT}. Note that nodes in $P(M_{i,c})$ are not guaranteed to be in reception channel c.

The key idea of BTS is to schedule collision-free transmissions layer by layer, and channel by channel. Take layer i for example, BTS consists of two steps:

1) $P(M_{i,c}) \rightarrow M_{i,c}$ sequentially for $c = 1, ..., k$;
2) $M_{i,c} \rightarrow L_{i,c}$ simultaneously for $c = 1, ..., k$;

We call $\bigcup_{c=1}^{k} M_{i,c}$ as layer-i *dominators*, and $P(\bigcup_{c=1}^{k} M_{i,c})$ as layer-i *connectors*. For step 1), we schedule transmissions channel by channel to avoid *same-node* collision, which means a node can be a parent of nodes in M_{i,c_1} and nodes in M_{i,c_2} ($c_1 \neq c_2$). Then we use ditance2-coloring of $P(M_{i,c})$ to achieve collision-free scheduling to cover[1] $M_{i,c}$. Distance-2 coloring method is widely used to schedule collision-free transmission to avoid *cross-node* collision, which means, if two nodes within two hops transmit at the same slot, there is a collision in common neighbors. For step 2), we also schedule collision-free transmissions in different channels simultaneously using distance2-coloring of $M_{i,c}$. The details are shown in Algorithm 1. It is easy to verify that the time complexity of BTS is $O(kn^3)$.

B. Performance Analysis

Then we give a theorem that proves the correctness of BTS and shows the upper bound of the latency given by BTS.

[1]In this paper, nodes are covered means nodes receive broadcast packets.

Algorithm 1: Basic Transmission Scheduling

Input: $G = (V, E)$, A, s
Output: T, $txTime$

$T_{SPT} \leftarrow$ SPT tree in G rooted s; $l \leftarrow$ depth of T_{SPT};
$L_i \leftarrow$ nodes at level i in T_{SPT}, $0 \leq i \leq l$;
$H_c \leftarrow \{u | u \in V$ and $A(u) = c\}$, $1 \leq c \leq k$;
for $i \leftarrow 1$ *to* l **do**
  for $c \leftarrow 1$ *to* k **do**
    $L_{i,c} \leftarrow H_c \cap L_i$; $M_{i,c} \leftarrow \emptyset$;
    for *each* $u \in L_{i,c}$ **do**
      if $N(u) \cap M_{i,c} = \emptyset$ **then**
        $M_{i,c} \leftarrow M_{i,c} \cup \{u\}$;
    $color_1(u) \leftarrow$ coloring by first to last ordering in $G^2[P(M_{i,c}) + M_{i,c}]$ for $u \in P(M_{i,c})$;
    $color_2(u) \leftarrow$ coloring by smallest-degree-last ordering in $G^2[L_{i,c}]$ for $u \in M_{i,c}$;
$T \leftarrow 0$;
for $i \leftarrow 1$ *to* l **do**
  for $c \leftarrow 1$ *to* k **do**
    $txTime(u, c) \leftarrow T + color_1(u)$ for $u \in P(M_{i,c})$;
    $T \leftarrow T + \max_v\{color_1(v) | v \in P(M_{i,c})\}$;
  for $c \leftarrow 1$ *to* k **do**
    $txTime(u, c) \leftarrow T + color_2(u)$ for $u \in M_{i,c}$;
  $T \leftarrow T + \max_{v,c}\{color_2(v) | v \in M_{i,c}\}$;
return T, $txTime$;

Theorem 2. *Algorithm BTS is correct, and provides a collision-free broadcast scheduling with latency at most* $(4k + 12) * l$.

Proof: For algorithm BTS, the transmissions are scheduled layer by layer. The transmissions in layer $i+1$ do not start until layer i ends. We consider layer i ($1 \leq i \leq l$). Assume nodes in L_i are not covered, and nodes in L_{i-1} are covered. For step 1), we color $P(M_{i,c})$ in layer $i - 1$ front-to-end by distance2-coloring, it is collision-free and can cover $M_{i,c}$. Furthermore, transmissions in different channel are sequential. Thus we can avoid *same-node* collisions. After that, nodes in $M_{i,c}$ are covered. For step 2), because $M_{i,c}$ is the maximal independent set of $L_{i,c}$, which is a dominating set of $L_{i,c}$, the smallest-degree-last distance2-coloring of $M_{i,c}$ guarantees that collision-free transmissions of $M_{i,c}$ can cover $L_{i,c}$. Also, the parallel transmissions in different channel are collision-free. Then all nodes in L_i are covered. Thus algorithm BTS is correct and collision-free.

We analyze the broadcast latency for layer i. The cross-layer transmissions from layer $i - 1$ to i are channel by channel. Hence we can consider a single channel c, and then multiple k. Note that $M_{i,c}$ is still an independent set in UDG. Hence a node u in $P(M_{i,c})$ can have at most four neighbors in $M_{i,c}$, because u has a parent in layer $i - 2$ in T_{SPT}, which is independent of nodes in $M_{i,c}$. Then the distance2-

coloring of $P(M_{i,c})$ uses at most four colors. Otherwise, if a node $u \in P(M_{i,c})$ has the five color, it means u shares five neighbors with nodes in $P(M_{i,c})\backslash\{u\}$, i.e., connecting five neighbors in $M_{i,c}$, which contradicts. Hence four time slots are enough for single-channel cross-layer transmission. For transmissions from $M_{i,c}$ to $L_{i,c}$, because $M_{i,c}$ is a maximal independent set, the smallest-degree-last ordering distance2-coloring of $M_{i,c}$ uses at most 12 colors [10], [16]. Since the same layer transmission can be in parallel, we do not need to multiple k. Hence, twelve time slots are enough. Given that our analysis applies from layer 1 to layer l, the overall broadcast latency is at most $(4k+12)*l$. In other words, BTS algorithm has approximation ratio of $4k+12$. ■

V. Enhanced Transmission Scheduling

In this section, we present an enhanced algorithm ETS, which has approximation ratio of $k+23$. We notice that BTS uses sequential channel transmissions in cross-layer, which is too conservative. Also, the strict constraint that the next layer cannot start transmissions before last layer ends cannot utilize the natural no-collision of multi-channel transmissions. Based on these two observations, we propose ETS.

A. Algorithm Description

ETS is a broadcast tree based algorithm. If u is the parent of w, u is responsible for transmitting packets to w collision-free. The formal description of constructing broadcast tree is shown in Algorithm 2. As stated in BTS, we have *connectors* to connect *dominators*. ETS differs from BTS mainly in the selection of *connectors*. BTS simply selects $P(M_{i,c})$ as *connectors*, and schedules transmissions in separate channel to avoid *same-node* collision. ETS selects *connectors* greedy, i.e., selecting parent nodes to cover maximal uncovered *connectors*. Note that here we select nodes from L_i. All selected nodes to cover $M_{i+1,c}$ are recorded in $P_{i+1,c}$. For *dominators* $M_{i,c}$ to cover $L_{i,c}$, it is similar.

The broadcast scheduling is shown in Algorithm 3. Though we still find available transmission slot channel by channel and layer by layer, we break the layered transmission constraint. In other words, layer $i+1$ can start before layer i ends only if nodes in layer $i+1$ do not bring collisions to already scheduling. Let step 1) be $P(M_{i,c}) \rightarrow M_{i,c}$, and step 2) be $M_{i,c} \rightarrow L_{i,c}$ Note that for step 1) and 2) transmissions, we have $P_{i,c}$ and $M_{i,c}$ recorded respectively. Hence, we first select tx node u from $P_{i,c}$ ($M_{i,c}$) sequentially, and then select the minimum time t larger than reception time to satisfy no-collision constraints: (1) u does not bring collisions to already scheduled transmissions in common neighbors; (2) u can not transmit at the same slot that it has been assigned to other channels. The collision slots are recorded in $I(u)$. After scheduling all nodes in V, we can find the maximal transmission time T. From Algorithm 2 and 3, we can find that time complexity of ETS is also $O(kn^3)$.

B. Performance Analysis

We first give a lemma about the correctness of ETS.

Algorithm 2: Broadcast Tree Construction

Input: $G(V,E), l, L_{i,c}$
Output: T_b
for *each* $v \in V$ **do**
  $p(v) = \emptyset$;
for $i \leftarrow 1$ *to* l **do**
  for $c \leftarrow 1$ *to* k **do**
    while $\exists w \in L_{i,c}$ *s.t.* $p(w) = \emptyset$ **do**
      $u \leftarrow argmax_u|\{w \in N(u) \cap L_{i,c}|p(w) = \emptyset, u \in L_{i,c}\}|$;
      $C(u) \leftarrow \{w|w \in N(u) \cap L_{i,c}, p(w) = \emptyset\}$;
      for *each* $v \in C(u)$ **do**
        $p(v) \leftarrow u$;
      $M_{i,c} \leftarrow M_{i,c} \cup \{u\}$;
    while $\exists w \in M_{i+1,c}$ *s.t.* $p(w) = \emptyset$ **do**
      $u \leftarrow argmax_u|\{w \in N(u) \cap M_{i+1,c}|p(w) = \emptyset, u \in L_i\}|$;
      $C(u) \leftarrow \{w|w \in N(u) \cap M_{i+1,c}, p(w) = \emptyset\}$;
      for *each* $v \in C(u)$ **do**
        $p(v) \leftarrow u$;
      $P_{i+1,c} \leftarrow P_{i+1,c} \cup \{u\}$;
$V_b \leftarrow V$; $E_b \leftarrow \{(u,v)|u = p(v)\}$;
return $T_b = (V_b, E_b, C)$;

Lemma 1. *Algorithm ETS is correct and provides a collision-free broadcast scheduling.*

Proof: ETS uses no-collision rule to select transmission slot, so it must be collision-free. We only need to prove ETS provides a broadcast scheduling. Assume all nodes in L_i ($1 \leq i < l$) are covered now. We show that after transmission of $P_{i+1,c}$ and $M_{i+1,c}$ ($1 \leq c \leq k$), all nodes in L_{i+1} are covered. For any node $u \in L_{i+1}$, u must belong to a particular set $L_{i+1,c}$. If $u \in M_{i+1,c}$, it is covered by $P_{i+1,c}$. If $u \in L_{i+1,c}\backslash M_{i+1,c}$, it must be covered by $M_{i+1,c}$. Because nodes in $P_{i+1,c}$ transmit before nodes in $M_{i+1,c}$, nodes in L_{i+1} are fully covered. The proof is complete. ■

Let t_i be the time that all tx nodes in L_i finish their transmissions, $0 \leq i \leq l$. We can give following lemmas.

Lemma 2. *Let* $M_i = \bigcup_{c=1}^{k} M_{i,c}$. *For any* $u \in M_i$, $\max_c\{txTime(u,c)\} \leq t_{i-1}+20$ *where* $u \in M_{i,c}$, $1 \leq c \leq k$.

Proof: Note that u must be in some $M_{i,c}$, since we select $M_{i,c}$ by channel c. In other words, $M_{i,c_1} \cap M_{i,c_2} = \emptyset$ for $c_1 \neq c_2$. Given any channel c, for any node $u \in M_{i,c}$, $rcvTime(u) \leq t_{i-1}$, because $p(u) \in L_{i-1}$. Note that all interfering nodes of u are also in $M_{i,c}$. Let $N_{IS}(u,2)$ denote the set of nodes consisting of an independent set within two hops from any node u. $|N_{IS}(u,2)| \leq 19$ for UDGs [11]. In other words, for any $u \in M_{i,c}$, the maximal number of interfering nodes is 19, because $M_{i,c}$ is an independent set of

Algorithm 3: Enhanced Transmission Scheduling

Input: $G = (V, E), T_b, l, M_{i,c}, P_{i,c}$
Output: T, $txTime$

for *each* $v \in V$ **do**
  $rcvTime(v) \leftarrow \infty$;
  for $c \leftarrow 1$ *to* k **do**
    $txTime(v, c) \leftarrow 0$;
$rcvTime(s) \leftarrow 0$;
for $i \leftarrow 1$ *to* l **do**
  for $c \leftarrow 1$ *to* k **do**
    for $j \leftarrow 1$ *to* $|P_{i,c}|$ **do**
      $u \leftarrow j$-th node in $P_{i,c}$;
      $I_1(u) \leftarrow \{t|\exists w \in N(u)$ that receives a message coll-free at time $t\}$;
      $I_2(u) \leftarrow \{t|t = txTime(u, ch) > 0, ch \neq c\}$;
      $txTime(u, c) \leftarrow \min\{t|t > rcvTime(u)$ and $t \notin I_1(u) \cup I_2(u)\}$;
      for *each* $v \in C(u)$ **do**
        $rcvTime(v) \leftarrow txTime(u, c)$;
    for $j \leftarrow 1$ *to* $|M_{i,c}|$ **do**
      $u \leftarrow j$-th node in $M_{i,c}$;
      $I_1(u) \leftarrow \{t|\exists w \in N(u)$ that receives a message coll-free at time t$\}$;
      $txTime(u, c) \leftarrow \min\{t|t > rcvTime(u)$ and $t \notin I_1(u)\}$;
      for *each* $v \in C(u)$ **do**
        $rcvTime(v) \leftarrow txTime(u, c)$;
$T \leftarrow \max_{u,c}\{txTime(u, c)\}$ for $u \in V$, $1 \leq c \leq k$;
return T, $txTime$;

G. Hence, $txTime(u, c)\} \leq t_{i-1} + 20$. Because our argument is for any channel, this completes the proof of Lemma 2. ■

Lemma 3. *Let* $P_i = \bigcup_{c=1}^{k} P_{i,c}$*. For any* $u \in P_i$, $\max_c\{txTime(u, c)\} \leq t_{i-1} + k + 23$ *where* $u \in P_{i,c}$, $1 \leq c \leq k$.

Proof: For ETS, u is dominated by some node in M_i. Due to Lemma 2, $rcvTime(u) \leq t_{i-1} + 20$. Let $I(u)$ be the set of interfering slots. $|I(u)|$ must be less than the number of interfering nodes. For u, the collision includes *cross-node* collision $I_1(u)$ and *same-node* collision $I_2(u)$. Assume $u \in P_{i,c}$. u is responsible for transmitting packets to $M_{i+1,c}$. Hence, $|I_1(u)| \leq N(u) \cap M_{i+1,c} - 1$. Because u can connect at most five neighbors in independent set of UDGs, and u has a parent in layer $i - 1$ in T_{SPT}, $|I_1(u)| \leq 3$. It is trivial that $|I_2(u)| \leq k - 1$ since we only have k channels. Though u can be in P_{i,c_1} ($c \neq c_1$), this constraint holds for $u \in P_{i,c_1}$. Because $u \in P_{i,c}$ for $1 \leq c \leq k$ selects transmission slots greedy, we only need to consider the maximal constraint for all channels. Therefore, $txTime(u, c) \leq t_{i-1} + 20 + (3 + k - 1) + 1 = t_{i-1} + k + 23$. The proof is accomplished. ■

Combining Lemma 1 and 3, we can have a theorem.

Theorem 3. *Algorithm ETS is correct, and provides a collision-free broadcast scheduling with approximation ratio of* $k + 23$.

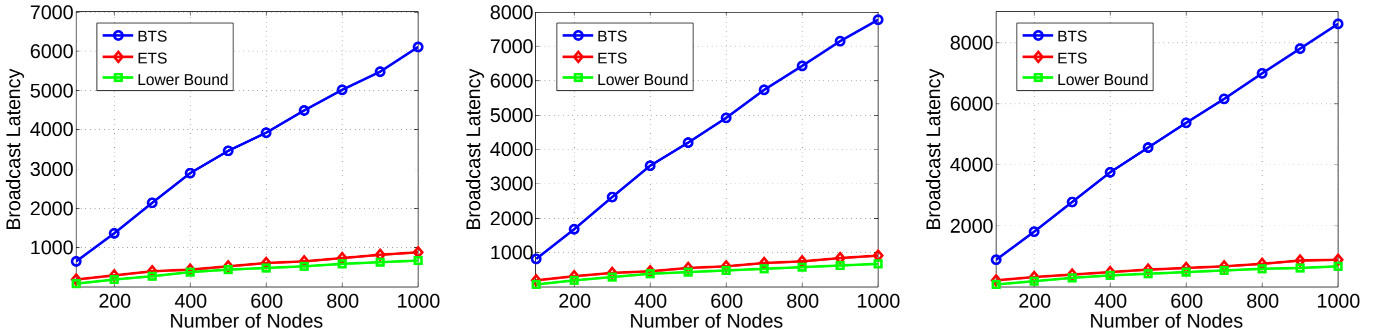
Proof: For source s, it needs at most k time slot, i.e. $t_0 \leq k$. Nodes in T_i transmit, and then nodes in PN_i transmit ($1 \leq i < l$). Finally, nodes in T_l transmit, and at most 20 time slots are enough. Hence, the overall latency is at most $k+(k+23)*(l-1)+20 \leq (k+23)*l$. Thus, the approximation ratio is $k + 23$. ■

VI. Performance Evaluation

In this section, we run simulations to study the performance of ETS. Since there is no directly applicable algorithms in SR-MC WANETs, we use BTS as benchmark. The metric is broadcast latency. To show the optimal result, we also plot the lower bound of broadcast latency using the depth of T_{SPT}. We consider the impact of number of nodes n and number of orthogonal channels k. To increase the depth of constructed T_{SPT}, we vary the network area with respect to n. For example, when $n = 1000$, the area size is $1000 \times 1000\ m^2$. All nodes are randomly deployed in corresponding areas, and their reception channels are randomly choosen from C. We run the simulation 10 times, and show the average results. For each time, we generate a new topology and channel assignment.

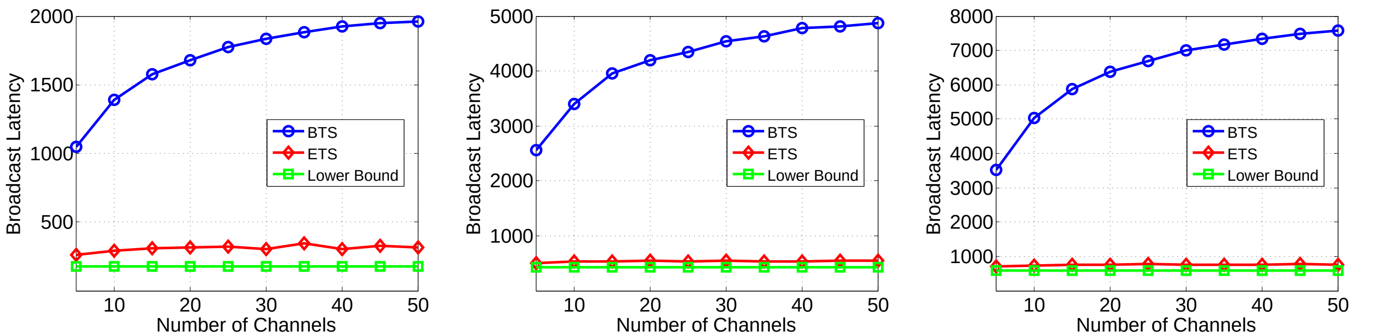
First we evaluate the impact of n, which ranges from 100 to 1000 with step 100. Simulation results with different $k = 10, 20, 30$ are shown in Figure 1. It is obvious that ETS performs better than BTS. More importantly, the performance of ETS is close to the lower bound, which demonstrates the gain of parallel multi-channel transmissions. Furthermore, when k becomes larger, the broadcast latency also raises due to the more nodes in each channel set, but the linear tendency keeps. Note that, for approximation ratio, the performance of both algorithms are much smaller than theoretical results. It can be explained that our theoretical analysis considers the worst case, but probably we are not in the worst case.

Then we study the impact of k, which is set from 5 to 30 with step 5. Simulation results with $n = 200, 500, 800$ are shown in Figure 2. The performance of ETS is still better than BTS, and close to the lower bound due to the same reason mentioned above. Note that the scale of y-axis in Figure 2(a) is smaller. Here, the depth of T_{SPT} keeps almost constant since n does not change (i.e., network size does not change). However, for BTS, the broadcast latency grows sub-linearly with respect to k. Intuitively, with larger k and same n, though the number of sequential transmissions raise linearly with respect to k, the transmissions in each channel reduce since nodes in each channel are less. For Figure 1 with larger n and constant k, transmissions in single-channel increase, but the number of sequential channel transmissions is the same. It can explain the linear and sub-linear phenomenon in Figure 1 and 2.



(a) (b) (c)

Fig. 1. Broadcast latency vs. network size n when (a) $k = 10$; (b) $k = 20$; (c) $k = 30$;



(a) (b) (c)

Fig. 2. Broadcast latency vs. number of channels k when (a) $n = 200$; (b) $n = 500$; (c) $n = 800$;

VII. Conclusion

In this paper, we consider the minimum latency broadcast scheduling problem in SR-MC WANETs. We first identify the challenge and opportunity in such networks. To solve the NP-hard problem, we give an algorithm BTS with approximation ratio of $4k + 12$, which is modified from classical algorithms. Then we propose an algorithm ETS with approximation ratio of and $k + 23$. Both have time complexity $O(kn^3)$. The simulation results show ETS improves the performance over BTS significantly, and come close to the lower bound. In the future, we want to complete our works by considering distributed algorithms, and finding algorithms for broadcast scheduling under *dynamic* channel assignment strategy.

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